

## Master Thesis

### Estimation of Unsteadiness in Model Parameters with Approximate Bayesian Computation

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## Abstract

In the present study, we attempt to estimate the unsteadiness of model parameters included in a physical model using the Approximate Bayesian Computation (ABC) method. In the ABC method, the parameter vector is given as random numbers according to a proposal distribution, and measured data and simulated data are directly compared to obtain the posterior distribution. The ABC method is an approximate method of estimating the parameter vector without calculating the likelihood. In particular, we propose the method to estimate the time variation of parameter vector. In this method, a set of consecutive data points is extracted from measured data and data points in the set are shifted step by step according to the time marching.

The estimation of aerodynamic coefficients of an obstacle in flight is considered as examples of physical model with unsteady model parameters. The numerical data of the trajectories of table tennis and soccer balls are employed as the measured data, which were obtained from flight images captured by video cameras.

In the ABC method, the distance function is defined as the distance between the measured and simulated data. The parameter vector used in computation of the simulated data is accepted when the value of the distance function is smaller than a tolerance. This means that the ABC method requires an appropriate tolerance for judging the accurate solution. For this reason, we propose the improvements for determining the tolerance.

Appropriate values of the tolerance are rather different in several trajectory data of different flight distances. Therefore, we employ the distance function nondimensionalized by flight distance of measured data. This improvement makes possible the estimation for trajectories data of different flight distances with the tolerance of the same order of magnitude .

Furthermore, we also propose the improvement to obtain the suitable tolerance by using the minimum value of the distance function. The minimum value is calculated by the rejection sampling method which is the simplest algorithm in the ABC method. The tolerance

is used by multiplying a constant to this minimum value, and then it is proportional to the measurement error in measured data. This improvement enables the automatic setting of a tolerance, and the results show that the accuracy of the estimation becomes better when measurement error in measured data is smaller.

On the basis of the above improvements, we estimate the time variation of the aerodynamic coefficients. The 10 and 30 consecutive data points are selected from the measured trajectory data. The results show that the same tendency in the temporal change of drag and lift coefficients although the result estimated in the 10 data points shows more detailed variation in the aerodynamic coefficients. However, in the estimation of the fewer data points the wider posterior distribution is obtained, which means that the estimation accuracy becomes lower.

The length of trajectories in projectiles increases with the flight time. Therefore, the deviation between the measured data and simulated data of incorrect parameters becomes small when the flight time is not sufficiently large. In such measured data, the parameter vectors far from the correct solution are more likely to be accepted. As a result, it is considered that the width of the posterior distribution increases.

Then, we define the distance function divided by  $t^{1.75}$  since this quantity keeps almost constant in time. However, the results with this distance function cannot improve the estimation accuracy in time variation of the model parameters. The division by the time length  $t^{1.75}$  may amplify the measurement error originally included in the trajectory data. It is therefore necessary to improve the accuracy in the estimated posterior distribution by taking the effect of time length in measured data into account.